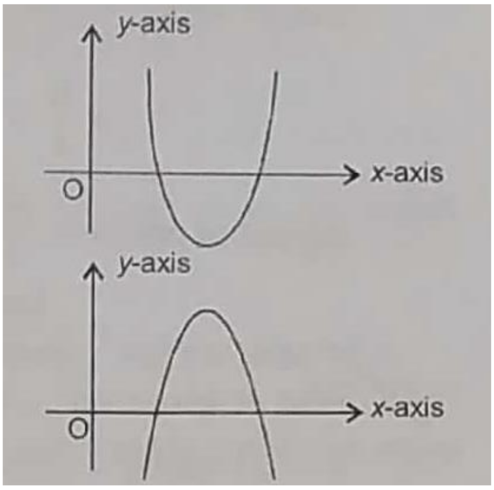
SUBJECT – MATHEMATICS
CLASS – X
SEASON-2020-2021
CHAPTER - 2

TOPIC - POLYNOMIALS

Basic Concepts

- Zeroes of a polynomial. K is said to be zero of a polynomial $p(x)\ if\ p(k) = 0$
- Graph of polynomial.



i. Graph of a linear polynomial $ax + b$ is a straight line.
ii. Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open upwards like U, if a>0.
iii. Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open downwards like ∩ $, if < 0$.
iv. in general a polynomial $p(x)$ of degree n crosses the x-axis at atmost n points.

Relationship between the zeroes and the coefficient of a polynomial.

i. If α, β are zeroes/roots of $p(x) = ax^2 + bx + c, a \neq 0$, then (refer NCERT for proof)
Sum of roots $=\alpha + \beta = \frac{-b}{a} \Rightarrow \alpha + \beta = \frac{-(coefficient\ of\ x)}{coefficient\ of\ x^2}$
Product of roots $=\alpha\beta = \frac{c}{a} \Rightarrow \alpha\beta = \frac{constant\ term}{coefficient\ of\ x^2}$
ii. If $\alpha, \beta\ and\ \gamma$ are zeroes / roots of $p(x) = ax^3 + bx^2 + cx + d, a \neq 0$ then, sum of roots $= \alpha + \beta + \gamma = \frac{-b}{a} = \frac{-(coefficient\ of\ x^2)}{coefficient\ of\ x^3}$. (refer RD Sharma for proof)

Sum of product of roots taken, two at a time $\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} = \frac{coefficient\ of\ x}{coefficient\ of\ x^3}$

Product of roots $= \frac{-d}{a} \Rightarrow \alpha\beta\gamma = \frac{-(constant\ term)}{coefficient\ of\ x^3}$

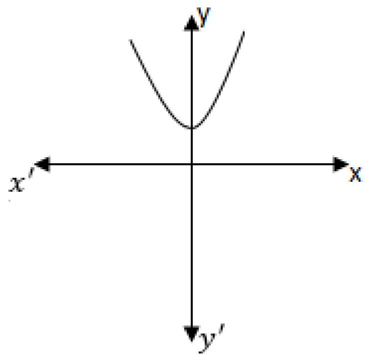
iii. if α, β are roots of a quadratic polynomial $p(x), then\ p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$
$\Rightarrow$ $p(x) = x^2 - (sum\ of\ roots)x + product\ of\ roots$

iv. if $\alpha, \beta\ and\ \gamma$ are zeroes of a cubic polynomial $p(x)$
Then, $p(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \alpha\gamma)x - (\alpha\beta\gamma)$
$\Rightarrow \quad p(x) = x^3 - (sum\ of\ zeroes)x^2 + (sum\ of\ product\ of\ zeroes/$
$roots\ taken\ two\ at\ a\ time)\ x\ - (product\ of\ zeroes)$

Practice Questions

1. Write the zeroes of polynomial $x^2 - x - 6$.
2. Write a quadratic polynomial, the sum and product of whose zeroes are 3 and –2.
3. Is$x = -3$, solution of the equation $2x^2 + 5x + 3 = 0$?
4. If $x + a$ is a factor of $2x^2 + 2ax + 5x + 10$, find a.
5. Show that $x = -2$ is a solution of $3x^2 + 13x + 14 = 0$.

6. The graph of $y = f(x)$ is given, how many zeroes are there of $f(x)$?



7. For what vale of $k, (-4)$ is a zero of the polynomial $x^2 - x - (2k + 2)$?
8. If 1 is a zero of polynomial $p(x) = ax^2 - 3(a - 1) - 1$, then find the value of a.
9. Write the polynomial, the product and sum of whose zeroes are $-\frac{9}{2}$ and $-\frac{3}{2}$ respectively.
10. i.If α, β are the zeroes of the polynomial, such that $\alpha + \beta = 6$ and $\alpha\beta = 4$, then write the polynomial.
    ii. If α, β are the zeroes of polynomial $2x^2 + 7y + 5$, write the value of $\alpha + \beta + \alpha\beta$.
    iii. If one zero of the polynomial $x^2 - 4x + 1$ is $2 + \sqrt{3}$, write the other zero.
11. Find others zeroes of $2x^4 - 9x^3 + 5x^2 + 3x - 1$, if two of its zeroes are $2 + \sqrt{3}$ and $2 - \sqrt{3}$.
12. If the polynomial $6x^4 + 8x^3 - 5x^2 + ax + b$ is exactly divisible by the polynomial $2x^2 - 5$, then find the values of a and b.
13. If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$, then what will be the quotient and remainder?
14. On dividing the polynomial $4x^4 - 5x^3 - 39x^2 - 46x - 2$ by the polynomial $g(x)$, the quotient and remainder were $x^2 - 3x - 5$ and $-5x + 8$ respectively. Find $g(x)$.
15. Show that $\frac{1}{2}$ and $\frac{-3}{2}$ are the zeroes of the polynomial $4x^2 + 4x - 3$ and verify the relationship between zeroes and co-efficients of polynomial.
16. Find the value of b for which $(2x + 3)$ is a factor of $2x^3 + 9x^2 - x - b$.
17. Find the other zeroes of the polynomial $x^4 - 7x^2 + 12$ if it is given that two of its zeroes are $\sqrt{3}$ and $-\sqrt{3}$.
18. Obtain all other zeroes of the polynomial $x^4 - 3x^3 - x^2 + 9x - 6$, if two of its zeroes are $\sqrt{3}$ and $-\sqrt{3}$.
19. Divide $2x^4 - 9x^3 + 5x^2 + 3x - 8$ by $x^2 - 4x + 1$ and verify the division algorithm.
20. Given that $x - \sqrt{5}$ is factor of the polynomial $x^3 - 3\sqrt{5}x^2 - 5x + 15\sqrt{5}$, find, all the zeroes of the polynomial.
21. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by $(x^2 - 2x + k)$ the remainder comes out to be $x + a$, find k and a.

22. What must be subtracted or added to $p(x) = 8x^4 + 14x^3 - 2x^2 + 8x - 12$ so that $4x^2 + 3x - 2$ is a factor of $p(x)$?

23. If 1 and -1 are zeroes of polynomial $Lx^4 + Mx^3 + Nx^2 + Rx + p$. Show that $L + N + P = M + R = 0$

24. If $x + a$ is a factor of the polynomial $x^2 + px + q$ and $x^2 + mx + n$ prove that $a = \frac{n-q}{m-p}$

25. Find a cubic polynomial with the sum, sum of the product of its zeros taken two at a time and product of its zeroes are $3, \frac{-1}{2}, \frac{5}{4}$ respectively.

26. $\alpha, \beta \; and \; \gamma$ are zeroes of polynomial $x^3 + px^2 + qx + 2$ such that $\alpha \beta + 1 = 0$. Find the value of $2p + q + 5$.

27. α, β, γ are zeroes of cubic polynomial $kx^3 - 5x + 9. If \; \alpha^3 + \beta^3 + \gamma^3 = 27$, find the value of k

28. α, β, γ are zeroes of cubic polynomial $x^3 - 12x^2 + 44x + c$. If α, β, γ are in AP, find the value of c.

29. Two zeroes of cubic polynomial $ax^3 + 3x^2 - bx - 6$ are -1 and -2. Find the third zero and value of a and b.

30. α, β, γ are zeroes of cubic polynomial $x^3 - 2x^2 + qx - r, if \; \alpha + \beta = 0$ then show that $2q = r$.

Notes – Practice all the sums given in NCERT Exercise (including optional exercise).

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